Exercise Sheet 1: Applications of the Gillespie algorithm
Workshop: "Stochastic simulations in branching process"

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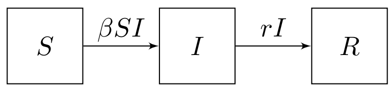
1 Gillespie algorithm - a simple birth-death process

We consider a population of identical individuals. Each individual reproduces at rate b and dies at rate d.

Simulate the population dynamics with the Gillespie algorithm with $b = 1.5$ and $d = 1$ in $t \in [0, 5]$. Compute the extinction probability starting with a single individual.

2 Several individual types - SIR model



Implement the above SIR model with the Gillespie algorithm with $\beta = 0.006$ and $r = 0.2$ in $t \in [0, 100]$. Compute the average number of recovered individual R at the end of the dynamics (i.e. the final epidemic size), starting with $S(0) = 100; I(0) = 1; R(0) = 0$.

3 Discrete time - Galton-Watson process (Optional)

We are modelling the growth of an epidemic in discrete time. Each generation corresponds to the infection period. Each infected individual generates a number of secondary cases drawn in a Poisson distribution with mean R_0.

Simulate the corresponding Galton-Watson process for $R_0 = 1.5$

4 A birth-death process with switching environment (Optional)

Consider two environments $\sigma = 0$ and $\sigma = 1$ that switch between them as

$$E_1 \xrightarrow{\lambda_-} E_0 \quad \text{and} \quad E_0 \xrightarrow{\lambda_+} E_1, \tag{1}$$

where E_σ represents the environmental state and $\lambda_\pm$ the switching rates. Consider a population coupled to these environments that undertakes a birth-death process with reactions

$$X \xrightarrow{b_\sigma} X + X \quad \text{and} \quad X \xrightarrow{d_\sigma} \emptyset, \tag{2}$$

where b_σ and d_σ are the birth and death rates, respectively, when in environment σ. Simulate this system using the Gillespie algorithm, using the following parameters: $b_0 = 0.3, d_0 = 0.1, b_1 = 0.1, d_1 = 0.2$ and an initial population size $n_0 = 10$. Plot n as function of t until a maximum time of $t = 100$. Assume that $\lambda_+ = \lambda_- = \lambda$ and explore the system dynamics for $\lambda = 0.1, 1, 10, 100$.