Two simple examples of the τ-leaping algorithm

Lotka-Volterra model of predator-prey interactions

$$A + B \xrightarrow{k_a} 2A$$
$$B \xrightarrow{k_b} 2B$$
$$A \xrightarrow{\mu} \emptyset$$

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$$\frac{da}{dt} = k_a ab - \mu a$$
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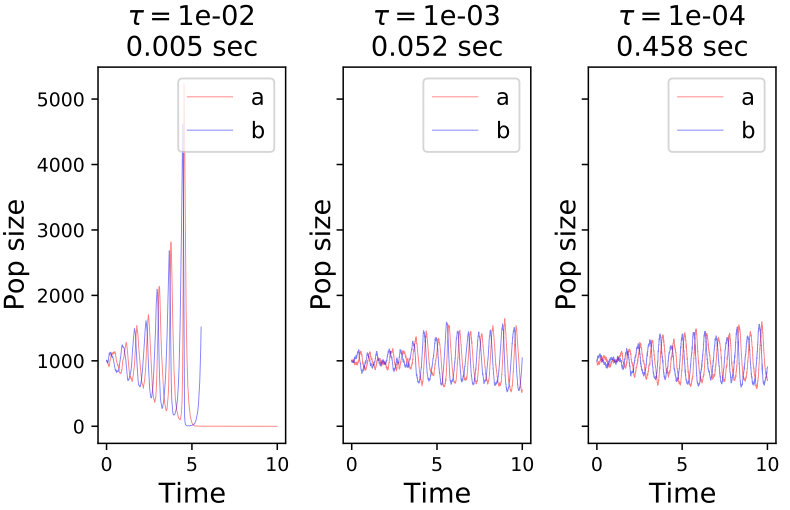
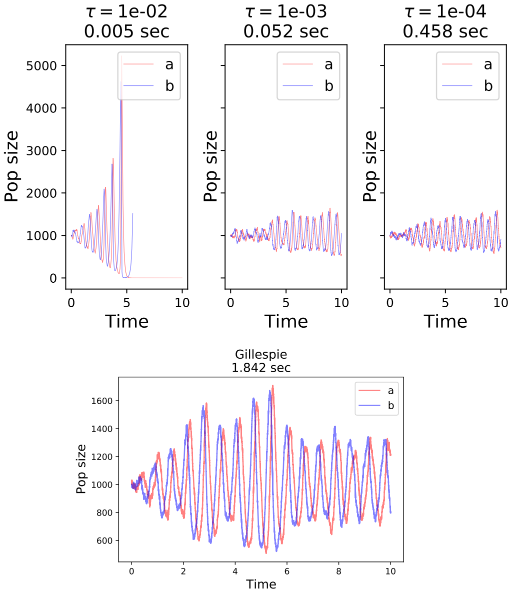
The impact of τ

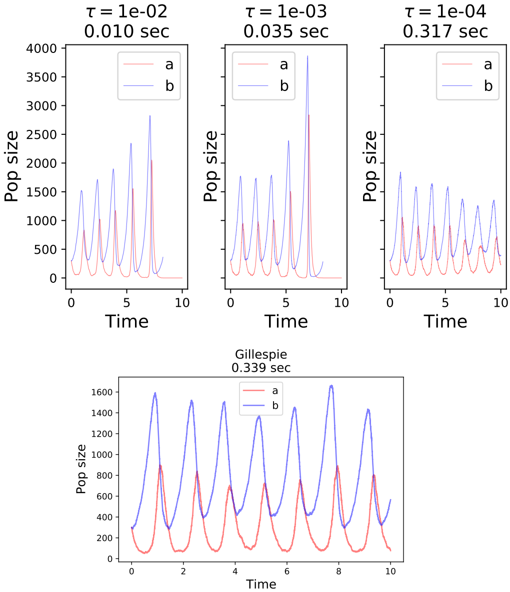


Large values of τ amplify the oscillations.

τ-leaping outperforms Gillespie for large populations



τ-leaping can be costly for small populations



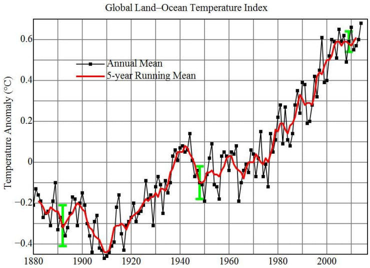
An example of gradual environment degradation

Time non-homogeneous birth-death process:

$$X \xrightarrow{b(t)} 2X$$

$$X \xrightarrow{d} \emptyset$$

$$\frac{dx}{dt} = (b(t) - d)x$$



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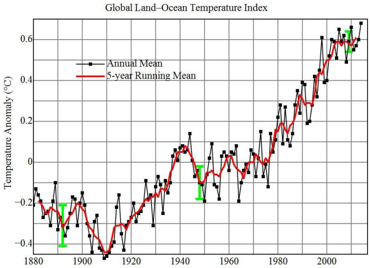

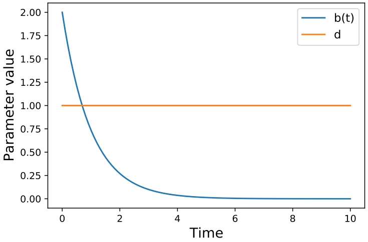

$$\sum_i \int_t^{t+\tau} a_i(x,s)\,ds = \ln\left(\frac{1}{r}\right)$$

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Temporal Gillespie algorithm

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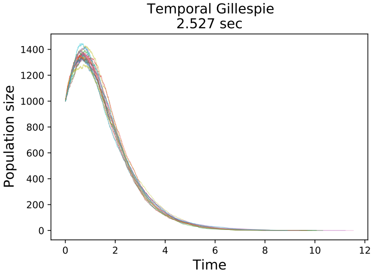

Lewis thinning algorithm

Several possibilities for the upper value of propensities:

$$b(t) \leq b_{max} \tag{1}$$

$$\tag{2}$$

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(2)

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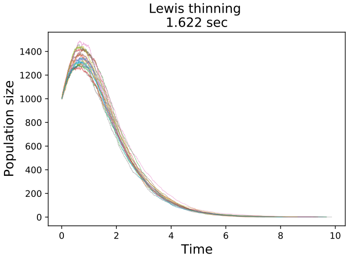

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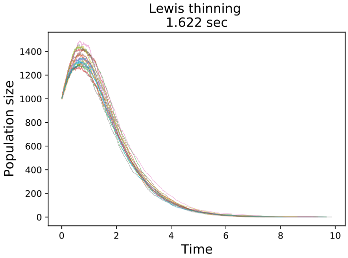

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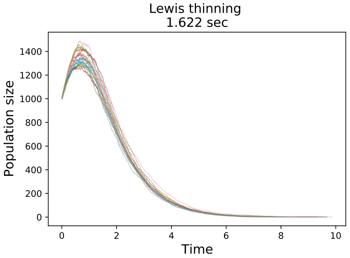

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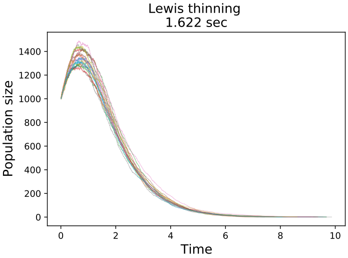

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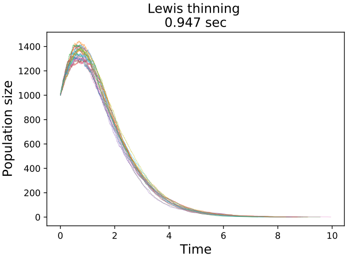
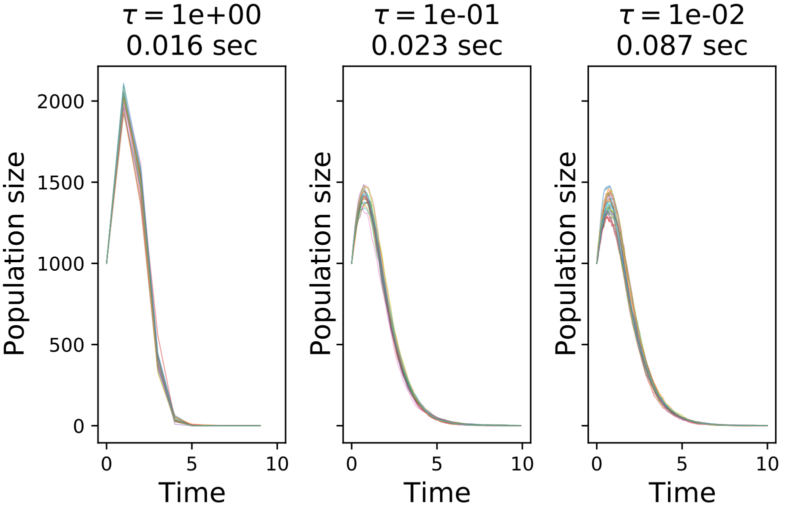


τ-leaping algorithm



If τ is too large, variation of $b(t)$ are not well taken into account.