Exercise Sheet 2: Applications of the $\tau-$leaping algorithm
Workshop: "Stochastic simulations in branching process"

Ernesto Berríos-Caro and Félix Geoffroy

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1 Single mutations

Consider a population of type S that mutate into a population of type A. Mutations occur through the offspring of S. The corresponding reactions are

1. $S \xrightarrow{b_S\cdot(1-\mu_A)} S+S$
2. $S \xrightarrow{d_S} \emptyset$
3. $S \xrightarrow{b_S\cdot\mu_A} S+A$
4. $A \xrightarrow{b_A} A+A$
5. $A \xrightarrow{d_A} \emptyset$

Simulate this system using the $\tau-$leaping algorithm for parameters $b_S = 1.0, d_S = 0.1, b_A = 1.5, d_A = 0.1, \mu_A = 10^{-7}$, for an initial condition of $n_S = 10^6$ and $n_A = 0$ at time $t = 0$. Estimate the average number of mutants A at time $t = 5$. Estimate this number for different values used for τ in the simulation. What would be an appropriate choice of τ for this system?

2 Decaying-dimerizing reaction set

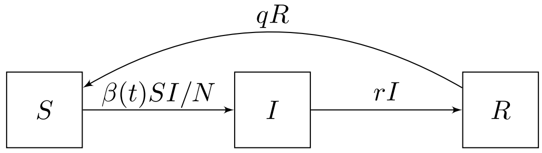
Consider a system composed by the following reactions:

1. $S_1 \xrightarrow{c_1} \emptyset$
2. $S_1+S_1 \xrightarrow{c_2} S_2$
3. $S_2 \xrightarrow{c_3} S_1+S_1$
4. $S_2 \xrightarrow{c_4} S_3$.

Simulate this system using the $\tau-$leaping algorithm. Estimate the average of the population sizes of S_1, S_2 and S_3 at time $t = 20$ for parameters $c_1 = 1, c_2 = 0.002, c_3 = 0.5, c_4 = 0.04$ for an initial condition $n_1 = 10^5$, $n_2 = n_3 = 0$ at time $t = 0$.

Hint: for reaction 2, notice that the propensity is $a_2 = c_2 \cdot n_1 \cdot (n_1 - 1)$.

3 Lewis thinning - periodical SIRS model (Optional)



The infection rate is time dependent $\beta(t) = \beta_0(1 + \beta_1 \cos(2\pi t))$. Note that $\beta(t) \leq \beta_0(1 + \beta 1) \quad \forall t$

Implement the above SIRS model with the Lewis thinning algorithm with $n = 5000$; $\beta_0 = 400$; $\beta_1 = 0.02$; $r = 40$ and $q = 0.5$ in $t \in [0, 10]$. Compute the average final number of infected $I(t = 10)$ conditioned on non-extinction, starting with $S(0) = N - 1; I(0) = 1; R(0) = 0$.